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INTRODUCTION TO THE MECHANICS OF A CONTINUOUS MEDIUM

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Preface

This book offers a unified presentation of the concepts and general principles common to all branches of solid and fluid mechanics, designed to appeal to the intuition and understanding of advanced undergraduate or first-year postgraduate students in engineering or engineering science.

The book arose from the need to provide a general preparation in continuum mechanics for students who will pursue further work in specialized fields such as viscous fluids, elasticity, viscoelasticity, and plasticity. Originally the book was introduced for reasons of pedagogical economy—to present the common foundations of these specialized subjects in a unified manner and also to provide some introduction to each subject for students who will not take courses in all of these areas. This approach develops the foundations more carefully than the traditional separate courses where there is a tendency to hurry on to the applications, and moreover provides a background for later advanced study in modern nonlinear continuum mechanics.

The first five chapters devoted to general concepts and principles applicable to all continuous media are followed by a chapter on constitutive equations, the equations defining particular media. The chapter on constitutive theory begins with sections on the specific constitutive equations of linear viscosity, linearized elasticity, linear viscoelasticity, and plasticity, and concludes with two sections on modern constitutive theory. There are also a chapter on fluid mechanics and one on linearized elasticity to serve as examples of how the general principles of the first five chapters are combined with a constitutive equation to formulate a complete theory. Two appendices on curvilinear tensor components follow, which may be omitted altogether or postponed until after the main exposition is completed.

Although the book grew out of lecture notes for a one-quarter course for first-year graduate students taught by the author and several colleagues during the past 12 years, it contains enough material for a two-semester course and is written at a level suitable for advanced undergraduate students. The only

prerequisites are the basic mathematics and mechanics equivalent to that usually taught in the first two or three years of an undergraduate engineering program. Chapter 2 reviews vectors and matrices and introduces what tensor methods are needed. Part of this material may be postponed until needed, but it is collected in Chap. 2 for reference.

The last 15 to 20 years have seen a great expansion of research and publication in modern continuum mechanics. The most notable developments have been in the theory of constitutive equations, especially in the formulation of very general principles restricting the possible forms that constitutive equations can take. These new theoretical developments are especially addressed to the formulation of nonlinear constitutive equations, which are only briefly touched upon in this book. But the new developments have also pointed up the limitations of some of the widely used linear theories. This does not mean that any of the older linear theories must be discarded, but the new developments provide some guidance to the conditions under which the older theories can be used and the conditions where they are subject to significant error. The last two sections of Chap. 6 survey modern constitutive theory and provide references to original papers and to more extended treatments of the modern theory than that given in this introductory text.

The book is a carefully graduated approach to the subject in both content and style. The earlier part of the book is written with a great deal of illustrative detail in the development of the basic concepts of stress and deformation and the mathematical formulation used to represent the concepts. Symbolic forms of the equations, using dyadic notation, are supplemented by expanded Cartesian component forms, matrix forms, and indicial forms of the same equations to give the student abundant opportunity to master the notations. There are also many simple exercises involving interpretation of the general ideas in concrete examples. In Chaps. 4 and 5 there is a gradual transition to more reliance on compact notations and a gradual increase in the demands on the reader's ability to comprehend general statements.

Until the end of Sec. 4.2, each topic considered is treated fairly completely and (except for the brief section on stress resultants in plate theory) only concepts that will be used repeatedly in the following sections are introduced. Then there begin to appear concepts and formulations whose full implementation is beyond the scope of the book. These include, for example, the relative description of motion, mentioned in Sec. 4.3 and also in some later sections, and the finite rotation and stretch tensors of Sec. 4.6, which are important in some of the modern developments referred to in the last two sections of Chap. 6. The aim in presenting this material is to heighten the reader's awareness that the subject of continuum mechanics is in a state of rapid development, and to encourage his reading of the current literature. The chapters on fluids and on elasticity also refer to published methods and results in addition to those actually presented.

The sections on the constitutive equations of viscoelasticity and plasticity are introduced by accounts of the observed responses of real materials in order to motivate and also to point up the limitations of the idealized representations that follow. The second section on plasticity includes work-hardening, a part of the theory not in a satisfactory state, but so important in engineering applications that it was believed essential to mention and point out some of the shortcomings of the available formulations.

A one-quarter course might well include most of the first five chapters, only part of Chap. 6, and either Chap. 7 on fluids or Chap. 8 on elasticity. Section 3.6 on stress resultants in plates and those parts of Secs. 5.3 and 5.4 treating couple stress can be omitted without destroying the continuity, as also can Secs. 6.5 and 6.6 on plasticity. Section 4.6 can be given only minor emphasis, or omitted altogether if the last two sections of Chap. 6 are not to be covered. The second appendix, presenting only physical components in orthogonal curvilinear coordinates might be included if time permits; although not needed in the text, it is useful for applications.

A two-term course could include the first appendix on general curvilinear tensor components, useful as a preparation for reading some of the modern literature. There is sufficient textual material in the book for a full year course, but it should probably be supplemented with some challenging applications problems. Most of the exercises in the text are teaching devices to illuminate the theory, rather than applications.

The book is a textbook, designed for classroom teaching or self-study, not a treatise reporting new scientific results. Obviously the author is indebted to hundreds of investigators over a period of more than two centuries as well as to earlier books in the field or in its specialized branches. Some of these investigators and authors are named in the text, but the bibliography at the end of the book includes only the twentieth-century writings cited. Extensive bibliographies may be found in the two *Encyclopedia of Physics* treatises; "The Classical Field Theories," by C. Truesdell and R. A. Toupin, Vol. III/1, pp. 226–793 (1960), and "The Non-Linear Field Theories of Mechanics," by C. Truesdell and W. Noll, Vol. III/3 (1965), published by Springer-Verlag, Berlin. These two valuable comprehensive treatises are among the references for collateral reading cited at the end of the introduction. Many of the historical allusions in the text are based on these two sources.

The author is indebted to several colleagues at Michigan State University who have used preliminary versions of the book in their classes. These include Dr. C. A. Tatro (now at the Lawrence Radiation Laboratory, Livermore, California) and Professors M. A. Medick, R. W. Little, and K. N. Subramanian. Professors John Foss and Merle Potter read the first version of the material on fluid mechanics. Encouragement and helpful criticism have been provided by these colleagues and also by the dozens of students who have taken the course.

The author is also indebted to Michigan State University for sabbatical leave during 1966–67 to work on the book and to Prentice-Hall, Inc., for their cooperation and assistance in preparing the final text and illustrations.

Finally, thanks are due to the author's wife for inspiration, encouragement and forbearance.

LAWRENCE E. MALVERN

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INTRODUCTION TO THE MECHANICS OF A CONTINUOUS MEDIUM

CHAPTER 1

Introduction

1.1 The continuous medium

The mechanics of a continuous medium is that branch of mechanics concerned with the stresses in solids, liquids, and gases and the deformation or flow of these materials. The adjective ***continuous*** refers to the simplifying concept underlying the analysis: we disregard the molecular structure of matter and picture it as being without gaps or empty spaces. We further suppose that all the mathematical functions entering the theory are continuous functions, except possibly at a finite number of interior surfaces separating regions of continuity. This statement implies that the derivatives of the functions are continuous too, if they enter the theory, since all functions entering the theory are assumed continuous. This hypothetical continuous material we call a ***continuous medium or continuum.***

The concept of a continuous medium permits us to define stress ***at a point***, a geometric point in space conceived as occupying no volume, by a mathematical limit like the definition of the derivative in differential calculus. This approach immediately makes the powerful methods of calculus available for the study of nonuniform distributions of stress, and at the same time provides an easily visualized physical model which agrees with the testimony of everyday observation of matter in the large. Thus this approach allows the mathematical analysis to be guided by intuition. The theories of elasticity, plasticity, and fluid mechanics based on the concept of continuous material lead moreover to quantitative predictions which agree closely with experience over a wide range of conditions. These theories and the further simplified engineering theories of beams and plates and shells, which are also based on the continuum concept of matter, are adequate for analysis of stress and deformation in most engineering problems.

There are exceptions of course. It would be too much to expect that a theory which disregards the molecular nature of matter could account for all

the observed properties of matter, even in the large. There is, for example, nothing in the theories that accounts for the formation of a fatigue crack in a body after many cycles of reversed loading. Problems in super aerodynamics at altitudes where the air is extremely rarefied may require molecular theories. Even in marginal situations like these, where continuum mechanics alone is inadequate, it is sometimes possible to use the continuum theory in combination with empirical information or with information derived from a physical theory based on the molecular nature of the material.

The use of the continuum concept to construct a working theory unifying a large body of observational knowledge and permitting the deduction of useful conclusions obviously has nothing to do with any assumption as to the real nature of matter. The existence of areas in which the theory is not applicable does not destroy its usefulness in other areas.

In most of the analyses in continuum mechanics, two further assumptions about the nature of the material are made, namely, that it is homogeneous and isotropic. It should be clearly understood that the three assumptions are completely independent.

Continuity. A material is continuous if it completely fills the space that it occupies, leaving no pores or empty spaces, and if furthermore its properties are describable by continuous functions.

Homogeneity. A homogeneous material has identical properties at all points.

Isotropy. A material is isotropic with respect to certain properties if these properties are the same in all directions.

It is quite possible, for example, to conceive of the atmosphere as a continuous fluid but still suppose that its density decreases with altitude. It is also possible to conceive of a rolled steel plate that is both continuous and homogeneous but has different tensile strengths or different elastic moduli in tension specimens cut from the plate, one with its axis in the direction of rolling and one with its axis at right angles to the direction of rolling. This kind of anisotropy can be introduced into continuum mechanics, and has led to useful results. The developments of the concepts of stress and strain in Chaps. 3 and 4 and the general principles of Chap. 5 use only the basic assumption of continuity. The other two simplifying assumptions are not needed until constitutive equations (stress-strain relations) or strength properties of a material are considered.

It is sometimes said that solutions to engineering problems obtained through the simplifying assumptions of the engineering theory of beams are only ap-

proximations to the "exact" solutions of the theory of elasticity. The foregoing paragraphs should have made it clear that the adjective "exact" is not justified. There may be exact mathematical solutions to the equations formulated in elasticity or in other branches of continuum mechanics, but the equations themselves are not exact descriptions of nature. In this respect the difference between the elementary theory and the advanced theory is one of degree rather than of kind. When the elementary theory is formulated consistently and logically, it is just as respectable as the advanced theory from a mathematical or logical point of view. And from a practical point of view it is just as good in those areas where its predictions agree closely enough with experience. The bounds of applicability of these elementary theories are determined by experience, either from experimental verification, or from comparisons with predictions of the more advanced continuum theories.

The last fifteen years have seen a spectacular growth in general continuum theory as well as in its specialized branches. The theory divides logically into three parts: (1) ***general principles***, assumptions and consequences applicable to all continuous media, (2) ***constitutive equations*** defining the particular idealized material, and (3) ***specialized theories*** of each individual idealized material built on the foundations of the general principles and the constitutive equations of that material to the point where boundary-value problems are formulated and solved for application to specific problems. The third part would require a book at least as long as this one for each specialized theory. Moreover, despite some superficial resemblance between the application of Laplace's equation in fluid mechanics and its application in some parts of linearized elasticity, there is very little common ground in the way the complete specialized theories are developed and applied. Hence specialized treatises must be consulted for the individual materials. Two small samples are included in Chap. 7 on fluid mechanics and Chap. 8 on linearized elasticity, to give some introduction to the way these two theories are put together and applied. References to specialized treatises on fluids and elasticity are given in those chapters. Some works on viscoelasticity and plasticity are also cited in the sections dealing with the constitutive equations of viscoelasticity and plasticity in Chap. 6. Until recently continuum mechanics was presented and studied almost exclusively in its separate specialized branches, but during the last ten years a definite trend has developed toward a unified treatment of the first two parts of the theory, the general principles and the constitutive theory.

The general principles are best studied within the framework of a general continuum theory. This approach emphasizes their general applicability, which is easily lost from sight when the general principles are interspersed with results applicable only to a specific theory. Some pedagogical economy is also achieved by presenting them only once instead of in two or three courses that might be taken by the same student. But an even more important advantage is

that the general principles can be presented more carefully, avoiding the tendency to hurry through them to get to the applications.

Most of the material in the first five chapters of this book, covering the general principles, is not new. Important concepts date back to Euler (1770) and much of the formal development was carried out during the early part of the nineteenth century by such pioneers as Cauchy, Navier, and Green, to mention only a few. The foundations of the theory have been subjected to extensive recent critical review. C. Truesdell (1952) published an important study, "The Mechanical Foundations of Elasticity and Fluid Mechanics," reprinted in book form in *Continuum Mechanics*, Vol. I (1966). Two monumental treatises, *Classical Field Theories* by Truesdell and R. Toupin (1960), and *Nonlinear Field Theories of Mechanics* by Truesdell and W. Noll (1965), have included comprehensive accounts of the foundations of the theory as well as of the newer developments in constitutive theory, with the most complete historical and bibliographical information on the subject up to that time. Because of the availability of these reference works no attempt will be made in this book to give a complete historical account, especially of the early work. For those early scientists mentioned in this text in connection with developments attributed to them, complete bibliographical information can be found in the three works cited above. These invaluable reference works and a few recent books at various levels on continuum mechanics are included in the list of references at the end of this chapter.

The most noteworthy recent developments in continuum mechanics have been in the theories of constitutive equations. The classical constitutive equations presented in the first six sections of Chap. 6 were developed independently before the advent of the modern theories of constitutive equations, although a few modifications to them have been made as a result of the modern theories. The modern theories are briefly referred to in Secs. 6.7 and 6.8, but their full development and application, especially to nonlinear constitutive equations, is beyond the scope of this introductory book. The modern theories are in essence theories of theories, setting some limitations on the forms the individual theories can take, once the proposed independent and dependent variables have been chosen. This reduces the amount of experimentation and intuition required to furnish the information needed for completing the definition of the individual idealized material. Historically the linear theories and the theory of plasticity developed by a different route, beginning with assumptions based on intuition and experience and only later being subjected to critical scrutiny to determine whether the form of the theory satisfies the requirements of a theory of theories. Some new constitutive equations will no doubt continue to spring up in this fashion, growing out of the needs of engineers and scientists. But the modern constitutive theories promise valuable aid in formulating new theories, especially of nonlinear material behavior.

Some of the pioneering contributions to the modern theories, especially the work of R. S. Rivlin and his associates and of W. Noll, dating from about 1956, are cited in Secs. 6.7 and 6.8. Many of the early papers and subsequent papers continuing the development of the theory have been reprinted in *Continuum Mechanics*, Vols. I, II, III, IV (1965–66), a four-volume series reprinting 49 papers published by 18 authors between 1945 and 1961, edited by C. Truesdell, referenced at the end of this chapter. The most complete account of the development growing out of the work of Noll is in the *Nonlinear Field Theories of Mechanics* already cited. For more introductory recent accounts see Jaunzemis (1967), Truesdell (1965), Leigh (1968), and Eringen (1967).

The general principles of Chap. 5 applicable to all continuous media, and constitutive equations defining the particular material, for example one of those given in Chap. 6, form the field equations of a continuum theory. The mechanical variables appearing in these equations are the stress and kinematic variables such as strain and rate of deformation. In elementary mechanics of materials, tensile strain is defined as elongation per unit length of the test specimen and is a dimensionless measure of the deformation. Stress is the internal force brought into play by the deformation, per unit cross-sectional area: it is equal to the externally applied force, per unit cross-sectional area. The stress tensor of a three-dimensional combined-stress field is a generalization of the elementary concept of stress, which will be developed in Chap. 3. The small-strain tensor of Secs. 4.1 and 4.2, a generalization of the elementary concept of strain, is the only kinematic variable appearing in the constitutive equation of linearized elasticity. But additional kinds of kinematic variables are needed for more general continua. Some of these additional kinematic concepts, including rate of deformation and finite strain and rotation tensors, will be developed in Chap. 4. The stress tensor will be treated first, since it is simpler and easier to visualize for most engineers, and it affords an opportunity to develop some skill in the use of tensors before the more complicated kinematic tensor development is encountered.

As a preparation for the stress tensor development of Chap. 3 and the kinematic tensor development of Chap. 4, a summary of vectors and tensors is given in Chap. 2, which can serve either as a quick review for the reader already acquainted with tensors or as a self-contained introduction to tensors. Additional material on curvilinear components of tensors is given in Appendices I and II, but these are not needed for the main body of the text. Some additional vector and tensor concepts are also presented where they are first needed in Chaps. 3, 5, and 7, and the reader to whom the tensor methods are new will find that the repeated use of them required throughout the book and the many cross references to Chap. 2 will greatly strengthen his understanding beyond that acquired in a first reading of Chap. 2.

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